## M122 College Algebra Review for Final Exam



**All answers should include your work (this could be a written explanation of the result, a graph with the relevant feature pointed out or an algebraic solution). On the Final Exam, less work = fewer points!! On final NO WORK = NO POINTS**

**Determine whether or not the relationship shown is a function. Explain why or why not.**

1)

| January | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Weight (lbs) | 210 | 209 | 211 | 210 | 209 | 208 | 207 |

Does the table define weight as a function of the day in January? Why or why not?

**Solve the problem.**

2) Suppose a cost–benefit model is given by $C(p) = \frac{1.5p}{100 - p}$, where C is the cost in thousands of dollars for removing p percent of a given pollutant. Find C(85) to the nearest dollar and interpret it.

3) Suppose the sales of a particular brand of appliance are modeled by the linear function $S(x) = 50x + 5800$, where S(x) represents the number of sales in year x, with x = 0 corresponding to 1982. Find the number of sales in 1996.

4) The population of a small U.S. town can be modeled by $P = -37t + 13{,}500$, where t is the number of years since 1975. Find and interpret all intercepts of the graph of this function.

5) A boat is moving away from shore after 1 hour it is 13.2 km from shore, after 4 hours it is 26.7 km from shore. What is the average rate of change of the distance from shore?

**Find the zero of f.**

6) $f(x) = 4x + 8$

**Solve the formula for the specified variable.**

7) $F = \frac{9}{5}C + 32$ for C

**Solve the problem.**

8) A gas station sells 4820 gallons of regular unleaded gasoline on a day when they charge $1.35 per gallon, whereas they sell 3917 gallons on a day that they charge $1.40 per gallon. Find a linear function that expresses gallons sold as a function of price. (Use the ordered pairs (1.35, 4820) and (1.40, 3917)

9) An average score of 90 for 5 exams is needed for a final grade of A. John's first 4 exam grades are 79, 89, 97, and 95. Determine the grade needed on the fifth exam to get an A in the course.

**Use the table to determine whether the data set represented is exactly linear, approximately linear, or nonlinear.**

10)

| x | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| y | 9 | 11 | 14 | 18 | 23 |

**Solve the problem.**

11) A shoe company will make a new type of shoe. The fixed cost for the production will be $24,000. The variable cost will be $37 per pair of shoes. The shoes will sell for $103 for each pair. Find a cost function *C(x)* for the total cost of producing *x* pairs of shoes and a revenue function *R(x)* for the total revenue generated. What is the profit if 600 pairs are sold?

12) At Allied Electronics, production has begun on the X–15 Computer Chip. The total revenue function is given by $R(x) = 43x - 0.3x^2$ and the total cost function is given by $C(x) = 10x + 12$, where x represents the number of boxes of computer chips produced. Find P(x).

13) Midtown Delivery Service delivers packages which cost \$1.70 per package to deliver. The fixed cost to run the delivery truck is \$80 per day. If the company charges \$6.70 per package, how many packages must be delivered daily to break even?

14) Find the market equilibrium point for the following supply and demand functions and explain what it means.
$D(p) = 790 - 12p$;
$S(p) = 390 + 3p$

15) Suppose that the sales of a particular brand of appliance satisfy the relationship $S(x) = 140x + 1900$, where $S(x)$ represents the number of sales in year x, with $x = 0$ corresponding to 1990. For what years will sales be between 2740 and 3160?

16) John owns a hot dog stand. He has found that his profit is given by the equation $P = -x^2 + 66x + 73$, where x is the number of hot dogs sold. How many hot dogs must he sell to earn the most profit?

**Find the coordinates of the vertex and determine whether the graph opens up or down.**

17) $y = (x + 8)^2 - 3$

18) $y = -4(x - 3)^2 - 4$

**Scale the axes and sketch the graph listing the vertex and all intercepts.**

19) $y = 60x^2 + 171x - 1275$

**Find the x–intercepts.**

20) $y = x^2 - 3x - 18$

**Use the quadratic formula to solve the equation and give the solution in simplest radical form (exact answers).**

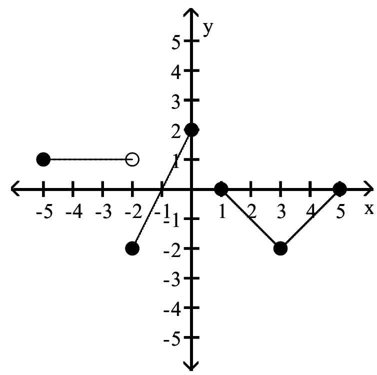
21) $7m^2 + 10m + 2 = 0$

**Solve the problem.**

22) The demand equation for a certain product is $P = 35 - 0.002x$, where x is the number of units sold per week and P is the price in dollars at which one is sold. The weekly revenue R is given by $R = xP$. What number of units sold produces a weekly revenue of \$33,000?

**Determine the intervals on which the function is increasing, decreasing, and constant.**

23)




**Write the equation of the graph after the indicated transformation(s).**

24) The graph of $y = x^2$ is shifted 2 units to the left. This graph is then vertically stretched by a factor of 5 and reflected across the x–axis. Finally, the graph is shifted 8 units downward.

**Solve the equation algebraically.**

25) $\sqrt{4x - 3} = 2x - 3$

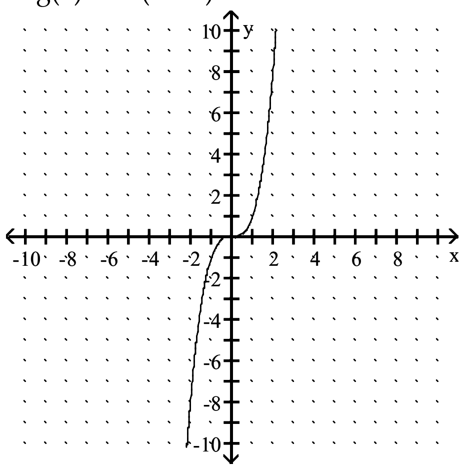
**Solve the problem.**

26) A study in a small town showed that the percent of residents who have college degrees can be modeled by the function $P = 35x^{0.35}$, where x is the number of years since 1990. According to the model, what percent of residents had a college degree in 1995? Round to the nearest whole number.

**Graph the function.**

27) $f(x) = \begin{cases} x - 2 & \text{for } x > 0 \\ -1 & \text{for } x \le 0 \end{cases}$

28) Given the graph of $f(x) = x^3$, draw the graph of $g(x) = -3(x - 3)^3 + 6$.



**Answer the question.**

29) How can the graph of $y = 0.6\sqrt[3]{-x} + 5$ be obtained from the graph of $y = \sqrt[3]{x}$

30) How can the graph of $f(x) = 0.5|-x| - 10$ be obtained from the graph of $y = |x|$?

**Find the requested value.**

31) A married couple's federal income tax in 2002 could be determined using the following function where x is the taxable income up to $109,250:

$f(x) =$

$$\begin{cases} 0.10x, \text{ if } x \le 12000 \\ 1200 + 0.15(x-12000), \text{ if } 12000 < x \le 45200 \\ 5280 + 0.28(x-45200), \text{ if } 45200 < x \le 109250 \end{cases}$$

What would the couples income tax be if thei taxable income was $35,000.

**For problems 47–51 find the new function and the domain.**

32) $f(x) = \sqrt{x-7}$ and $g(x) = \dfrac{1}{x - 8}$

Find (f g)(x) and its domain.

33) Given $f(x) = \dfrac{7}{x - 6}$ and $g(x) = \dfrac{5}{7x}$,

Find f (g(x)) and its domain.

34) For $f(x) = x^2 - 36$ and $g(x) = 2x + 3$

Find $\left(\dfrac{f}{g}\right)(x)$ and its domain.

**Solve the problem.**

35) The cost of manufacturing clocks is given by $C(x) = 67 + 40x - x^2$. Also, it is known that in t hours the number of clocks that can be produced is given by $x = 8t$, where $1 \le t \le 12$. Express C as a function of t.

**Determine whether or not the function is one–to–one and explain why or why not.**

36) This chart shows the number of hits for five Little League baseball teams.

| Team | Hits |
|---|---|
| Hawks | 28 |
| Lions | 41 |
| Eagles | 55 |
| Bears | 55 |
| Dolphins | 20 |

37) $f(x) = 4x^2 + x$

**Find the inverse of the function.**

38) $f(x) = x^3 - 3$

**Use algebraic and graphical or numerical methods to solve the inequality.**

39) $x^2 + 2x \le 3$

**Solve the problem.**

40) The cost C of producing t units is given by $C(t) = 4t^2 + 9t$, and the revenue R generated from selling t units is given by $R(t) = 5t^2 + t$. For what values of t will there be a profit?

41) A coin is tossed upward from a balcony 262 ft high ($h_o$) with an initial velocity ($v_o$) of 16 ft/sec, according to the formula $h(t) = -16t^2 + v_ot + h_o$, where t is time in seconds. During what interval of time will the coin be at a height of at least 70 ft?

42) The number of books in a small library increases according to the function $B = 3600e^{0.05t}$, where t is measured in years after opening.
a) How many books did the library have when they opened?
b) How many books will the library have 6 years after opening?

**Graph the function showing all asymptotes and intercepts.**

43) $f(x) = 4^{(x+1)} - 2$

44) $f(x) = \log_4 (x + 2)$

**Provide an appropriate response.**

45) Explain how the graph of $y = \left(\frac{1}{3}\right)^{(x-4)} + 5$ can be obtained from the graph of $y = 3^x$.

46) Explain how the graph of $y = -5\log_2(x+3) - 1$ can be obtained from the graph of $y= \log_2 x$.

**Write the logarithmic equation in exponential form.**

47) $\log_4 \frac{1}{64} = -3$

48) $\ln x = -6$

**Write the exponential equation in logarithmic form.**

49) $6^3 = 216$

50) $e^{4x} = y$

**Find the value of the logarithm without using a calculator.**

51) $\log_5 \sqrt{5}$

**Use a change of base formula to evaluate the given logarithm. Approximate to three decimal places.**

52) $\log_4 (0.654)$

**Find the inverse of the function.**

53) $f(x) = 2^{-x} + 3$

**Solve the problem.**

54) The logarithmic function $f(x) = -100 + 87 \ln x$ models the number of visitors (in millions) to U.S. museums from 1940 to 1990, where x is the number of years since 1900. Use this function to estimate the number of visitors in the year 1983.

**Provide an appropriate response.**

55) For the function: $f(x) = \log_{10} x$?
a) What is the domain of the function?
b) What is the range of the function?

**Solve the equation algebraically.**

56) $29^x = 89$ (Round to two decimal places.)

57) $50e^{-0.647x} = 125$(Round to three decimal places.)

58) $\log 4x = \log 5 + \log (x - 4)$

59) $\log_2(3x - 2) - \log_2(x - 5) = 4$

60) $5 \ln(x - 4) = 10$

**Rewrite the expression as the sum and/or difference of logarithms, without using exponents. Simplify if possible.**

61) $\log_4 \frac{x^8 y^9}{3}$

**Rewrite as a single logarithm.**

62) $(\log_t t - \log_t s) + 3 \log_t u$

**Solve the problem.**

63) Assume the cost of a car is \$16,000. With continuous compounding in effect, the cost of the car will increase according to the equation $C = 16{,}000e^{rt}$, where r is the annual inflation rate and t is the number of years. Find the number of years it would take to double the cost of the car at an annual inflation rate of 2.1%. Round the answer to the nearest hundredth.

64) In the formula $A(t) = A_0e^{kt}$, A is the amount of radioactive material remaining from an initial amount $A_0$ at a given time t, and k is a negative constant determined by the nature of the material. A certain radioactive isotope decays at a rate of 0.175% annually. Determine the half–life of this isotope, to the nearest year.

65) A bacteria culture starts with 10,000 bacteria, and the number doubles every 40 minutes.

a) Find the exponential growth function that models the number of bacteria after t minutes

b) Find the number of bacteria after one hour

c) After how many minutes will there be 50,000 bacteria?

66) A radioactive substance has a half–life of 140 days. Suppose a sample of this substance has a mass of 300 mg.

a) Find an exponential decay function that models the amount of the sample remaining after t days.

b) Find the mass remaining after one year.

c) How long will it take for the sample to decay to a mass of 200 mg?

67) Find the amount of money in an account after 9 years if $3500 is deposited at 4% annual interest compounded monthly.

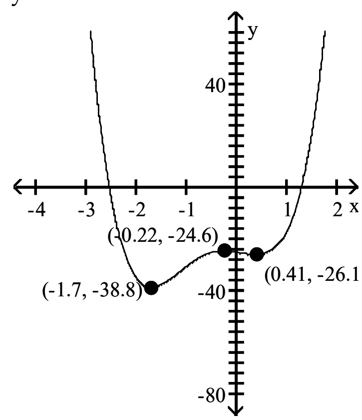
68) How long would it take $9000 to grow to $45,000 at 7% compounded continuously? Round your answer to the nearest tenth of a year.

69) Find the annual interest rate if $3500 is deposited in an account that compounds semi–annually and after 1.5 years the future value is $4000

70) The number of students infected with the flu on a college campus after t days is modeled by the function $P(t) = \dfrac{320}{1 + 39e^{-0.3t}}$. What is the maximum number of infected students possible?

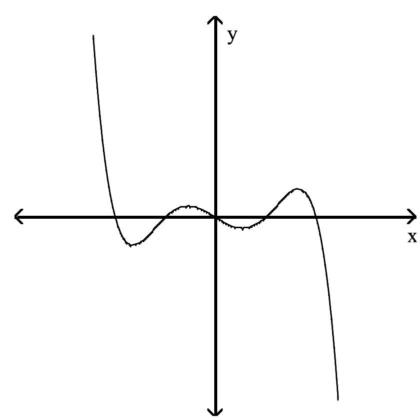
**Determine a. The interval where the graph is increasing**
**b. The interval where the graph is decreasing**
**c. The intercepts**
**d. The local maxima**
**e. The local minima**

71) $y = 5x^4 + 10x^3 - 4x^2 - 3x - 25$




**Determine the minimum degree of the polynomial and state whether the leading coefficient is positive or negative.**

72)

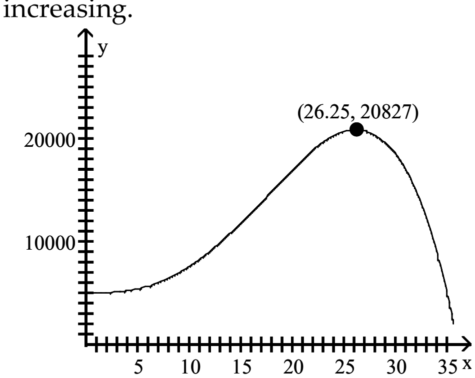


**Solve the problem.**

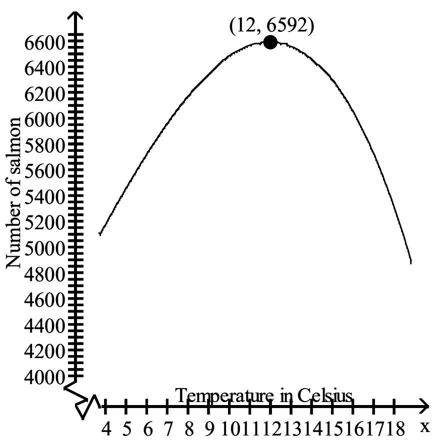
73) Suppose that the population of a certain city can be approximated by:

$$P(x) = -0.1x^4 + 3.5x^3 + 5000$$

where x is time in years since 1960. Using the graph of P(x) below,, estimate during what time period the population of the city was increasing.




74) $S(x) = -x^3 + 6x^2 + 288x + 4000$, $4 \le x \le 20$ is an approximation to the number of salmon swimming upstream to spawn, where x represents the water temperature in degrees Celsius. Using the graph below, find the temperature that produces the maximum number of salmon.




**Solve the polynomial equation by using the root method.**

75) $7x^3 - 189 = 0$

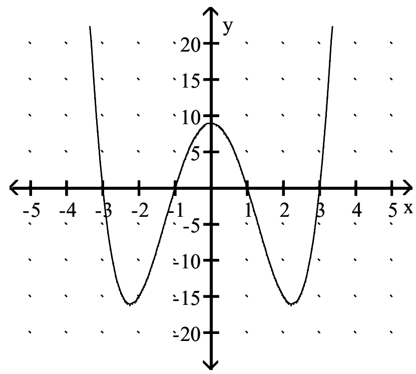
**Solve the polynomial equation by factoring.**

76) $x^3 - 100x = 0$

77) $2x^3 + 4x^2 - 2x - 4 = 0$

**Use the graph of the polynomial function f(x) to solve f(x) = 0.**

78)




**Solve the problem.**

79) If the price for a product is given by $p = 900 - x^2$, where x is the number of units sold, then the revenue is given by $R = px = 900x - x^3$. How many units must be sold to give zero revenue?

80) Suppose a business can sell x gadgets for $p = 250 - 0.01x$ dollars apiece, and it costs the business $c(x) = 1000 + 25x$ dollars to produce the x gadgets. Determine the production level required to maximize profit.

81) The Cool Company determines that the supply function for its basic air conditioning unit is $S(p) = 400 + 0.01p^2$ and that its demand function is $D(p) = 2500 - 0.2p^2$, where p is the price. Determine the price for which the supply equals the demand.

**Graph the function showing all asymptotes and intercepts.**

82) $f(x) = \dfrac{3x - 4}{x - 1}$

**For the given rational function f, find all values of x for which f(x) has the indicated value.**

83) $f(x) = \dfrac{x - 2}{x - 4}$;   $f(x) = 3$

**Use analytical methods to solve the equation.**

84) $\dfrac{40}{x - 2} = 1 + \dfrac{42}{x + 2}$

**Solve the problem.**

85) In the following formula, y is the minimum number of hours of studying required to attain a test score of x: $y = \dfrac{0.38x}{100.5 - x}$. How many hours of study are needed to score 90?

86) Suppose that a cost–benefit model is given by

$$f(x) = \frac{5.9x}{100 - x}$$

where f(x) is the cost in thousands of dollars of removing x percent of a given pollutant. What is the vertical asymptote of the graph of this function? What does this suggest about the possibility of removing all of the pollutant? Explain your reasoning.

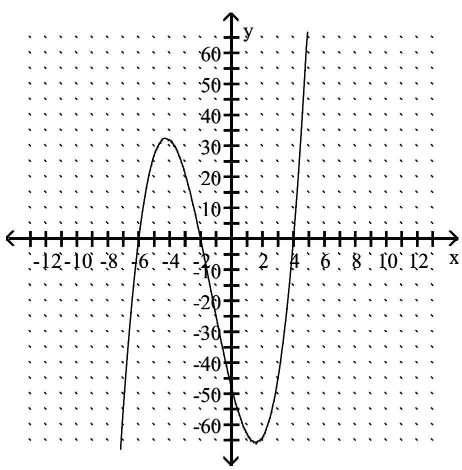
**Solve the inequality.**

87) $(x + 9)(x + 5)(x - 8) > 0$

88) $\dfrac{2x + 4}{5x^2 - 5} > 0$

**Use the graph of f to solve the inequality.**

89) $f(x) < 0$

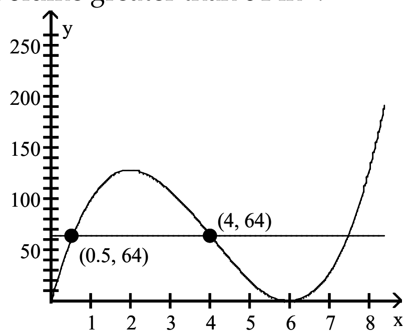



**Solve the problem.**

90) An open–top box is to be made by cutting small identical squares from each corner of a 12–by–12–in. sheet of tin and bending up the sides. If each corner square is x inches on a side, the volume of the box (in in.$^3$) is given by:

$$V(x) = 144x - 48x^2 + 4x^3$$

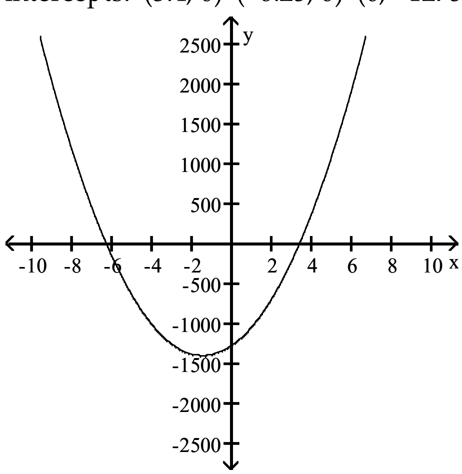
Using the sketch of the graph of V(x) below, estimate what values of x result in a box with volume greater than 64 in$^3$.




91) The monthly sales volume y (in thousands of dollars) is related to monthly advertising expenditures x (in thousands of dollars) according to the equation $y = \dfrac{100x}{x + 10}$.

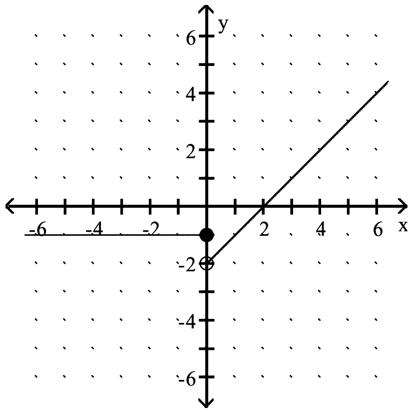
Spending how much money on advertising will result in sales of at least $75,000 per month?

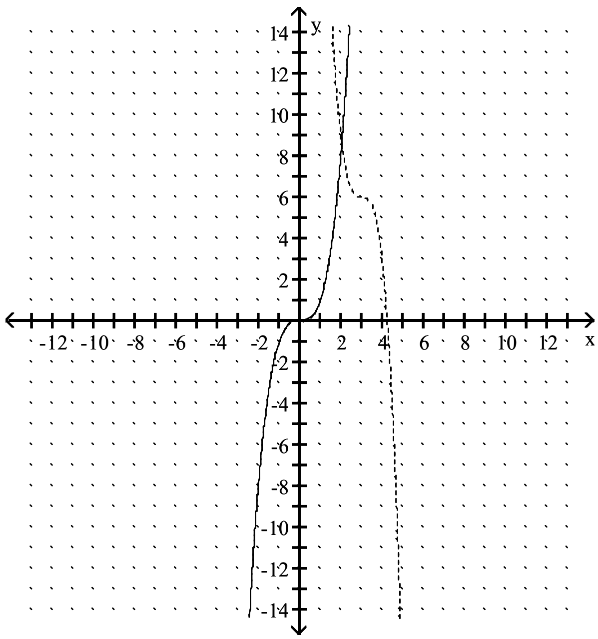
1) Yes. For each input value (day) there is exactly one output value (weight)
2) \$8500; It will cost \$8500 to remove 85% of the pollutant.
3) 6500
4) (0, 13500) : The population of the town was 13,500 in 1975.
(364.86, 0): If the population continues to decrease at the same rate, the population will be zero after 365 years (23
5) 4.5 km/hr
6) −2
7) $C = \frac{5}{9}(F - 32)$
8) G(p) = −18,060x + 29,201
9) 90
10) Nonlinear
11) C(x) = 24,000 + 37x; R(x) = 103x; \$15,600
12) $P(x) = -0.3x^2 + 33x - 12$
13) 16 packages
14) When the price is \$26.67, the amount demanded equals the amount supplied which is 470.
15) Between 1996 and 1999
16) 33 hot dogs
17) (−8, −3); concave up
18) (3, −4) Concave down
19) vertex: (−1.425, −1396,8375)
intercepts: (3.4, 0) (−6.25, 0) (0, −1275)




20) (6, 0) and (−3, 0)
21) $\frac{-5 \pm \sqrt{11}}{7}$
22) 1000 or 16,500
23) Increasing on (−2, 0) and (3, 5); Decreasing on (1, 3); Constant on (−5, −2)
24) $y = -5(x + 2)^2 - 8$
25) 3
26) 61%

27)




28)

29) Reflect it across the y−axis. Shrink it vertically by a factor of 0.6. Shift it vertically 5 upward.

30) Reflect it across the y−axis. Shrink it vertically by a factor of 0.5. Shift it vertically 10 units downward.

31) $4,650

32) $\frac{\sqrt{x-7}}{x-8}$; domain: $[7,8)\cup(8,\infty)$

33) $\frac{49x}{5-42x}$; domain: $\{x \mid x\neq 0,\ x\neq \frac{5}{42}\}$

34) $\frac{x^2-36}{2x+3}$; $\{x \mid x\neq \frac{-3}{2}\}$

35) $C(t) = 67 + 320t - 64t^2$

36) No, the output value of 55 is repeated with different input values.

37) No, its graph does not pass the horizontal line test.

38) $f^{-1}(x) = \sqrt[3]{x+3}$

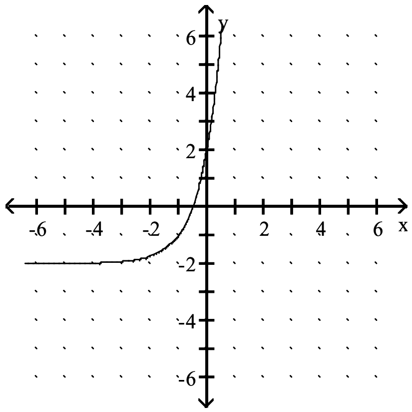
39) $-3 \le x \le 1$

40) $t > 8$

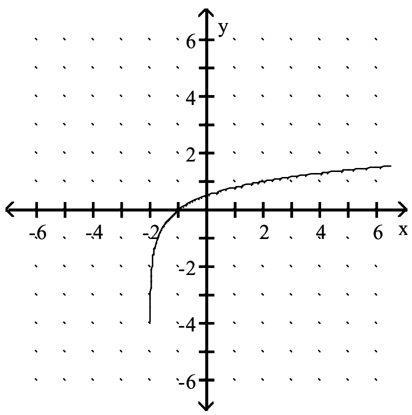
41) 0 sec $\le t \le$ 4 sec

42) 3600; 4860

43)




44)




45) The graph is reflected over the y−axis, shifted 4 units to the right and 5 units up.

46) The graph is shifted 3 units to the left, stretched by a factor of 5, reflected across the x−axis and shifted 1 unit down.

47) $4^{-3} = \frac{1}{64}$

48) $x = e^{-6}$

49) $\log_6 216 = 3$

50) $\ln y = 4x$

51) $\frac{1}{2}$

52) −0.306

53) $f^{-1}(x) = -\log_2 (x - 3)$

54) 284.4 million

55) a) $(0, \infty)$ b) $(-\infty, \infty)$

56) 1.33

57) −1.416

58) {20}

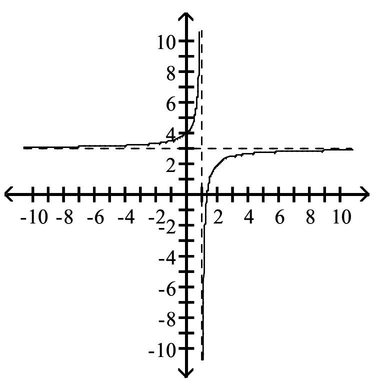
59) $x = 6$

60) $e^2+4 \approx 11.39$

61) $8 \log_4 x + 9 \log_4 y - \log_4 3$

62) $\log_t \frac{tu^3}{s}$

63) 33.01
64) 396 yr
65) a) $n(t) = 10000e^{0.01733t}$
    b) 28,287 bacteria
    c) 93 minutes
66) a) $m(t) = 300e^{-0.00495t}$
    b) 49 mg.
    c) 82 days
67) $5013.65
68) 23.0 years
69) 9.1%
70) 320
71) a. $(-1.7, -.22) \cup (-.41, \infty)$
    b. $(-\infty, -1.7) \cup (-.22, .4)1$
    c. $(0, -25)$, $(-2.5, 0)$, $(1.25, 0)$
    d. $-24.6$
    e. $-38.8$, $-26.1$
72) Minimum degree is 5. Leading coefficient is negative.
73) Between 1960 and 1986
74) 12°C
75) 3
76) 0, 10, −10
77) −2, 1, −1
78) −3, −1, 1, 3
79) 0, 30
80) 11,250 gadgets
81) $100.00
82)




83) 5
84) 12, −14
85) 3.26 hr
86) Answers will vary. Possible answer: The vertical asymptote is x = 100. This suggests that as x approaches 100 percent, the cost becomes infinitely large. It is therefore not possible to remove all of the pollutant.
87) $-9 < x < -5$ or $x > 8$
88) $(-2,-1)\cup(1,\infty)$
89) $x < -6$ or $-2 < x < 4$
90) 0.54 in. $\le x \le$ 4 in.
91) At least $30,000